A Linear Statistical FET Model Using Principal Component Analysis

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Abstract —**An important issue in statistical circuit design, other than the algorithms themselves, is the development of efficient, statistically valid element models. This paper first presents what is needed for a good statistical model. The standard FET model is shown to be difficult to use in a statistical simulation, due to the nonlinear relation between FET S parameters and model parameters. A linear statistical FET model is then proposed based upon principal component analysis. This linear model gives uncorrelated model parameters. In an example using measured S-parameter data from 90 0.5 μm GaAs FET's, 13 uncorrelated model parameters were needed to model the data from 1 to 11 GHz and at one bias. Simulation using this linear model and issues relating to bias are discussed.**

I. Introduction

With the aid of sophisticated computer-aided design (CAD) software, microwave circuit designs can be optimized for a certain performance criterion. These designs give optimum performance for a fixed set of circuit parameters (the nominal parameter values). However, when the parameter values are statistically perturbed from the nominal values, as happens during manufacturing, the circuit's performance is not analyzed or specified. Many authors have shown that a design optimized for good performance at a single set of parameter values can perform poorly when the parameter values are perturbed [1], [2]. The purpose of statistical circuit design is to determine nominal parameter values that give acceptable circuit performance when the parameter values are statistically perturbed.

The past focus in statistical circuit design has been design for high parametric yield, which is the fraction of circuits which meets specifications when the circuit parameters statistically vary around their nominal values. Presently, if the statistics of the circuit parameters are known and if valid statistical models are used, software tools exist which can determine circuit designs for which parametric yield is in some sense optimized [3], [4].

An important issue in statistical circuit design, other than algorithms, is the need for statistically valid and efficient element models. This apparently has not been discussed in the literature except in [5]. This work reported that FET model parameter statistics affect both the final values of the statistical circuit design and the estimated circuit yield. The accuracy in the results of the entire statistical circuit design and yield estimation process depends on the statistical representation of the circuit elements. This is especially true for accurate yield estimation.

A good statistical model for a circuit element must represent the important statistical properties of the element with a small number of parameters over the operating range (e.g. frequency and bias) of the element. This paper proposes a linear statistical FET model. Section II presents the FET S-parameter statistics for a GaAs FET and Section III addresses the statistical validity of a present nonlinear FET model. In Sections IV and V the proposed linear model is presented and the results of modeling a 0.5 μm GaAs FET are given. Section VIII presents conclusions.

II. FET S-Parameter Statistics

When a foundry characterizes the RF properties of a FET they take measurements of the FET's two-port S parameters. In general, if many FET's are measured, each set of measured S parameters will be different. Therefore, the S parameters can be modeled as random variables with a given joint density function which is estimable from the measured data. The goal of a FET statistical simulation is to create simulated S parameters which are valid samples of the measured S-parameter joint density function. To better understand the requirements for a valid statistical model, actual measured FET statistics gathered from 90 FET's will be presented.

The GaAs FET's were fabricated from January 1987 to June 1987 using a standard process of TriQuint Semiconductor Inc. [6]. The 0.5×300 μm^2 FET's are described in [5]. S-parameter data were taken on 90 FET's at two biases and frequencies from 1 to 26 GHz. A total of 90 FET's is a small sampling and as a result the density histograms are rough and not fully filled out. Fig. 1 shows the densities for the real part of $S11$ over the frequencies 1 to 11 GHz. These data are typical of all the FET densities. Note that these densities are not Gaussian. However it is

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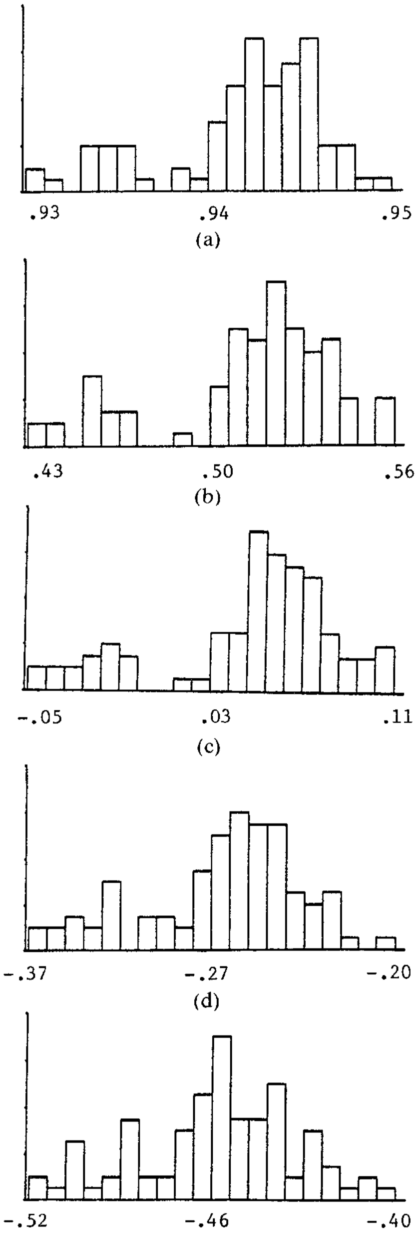

Fig. 1. Marginal densities for the real part of $S11$ over the frequencies (a) 1 GHz, (b) 3.5 GHz, (c) 6 GHz, (d) 8.5 GHz, and (e) 11 GHz.

possible that if these data were recorded over a shorter time interval, the densities would be closer to Gaussian.

Samples of the estimated correlation matrix are shown in Tables I and II. Table I shows the 8×8 correlation matrix of the measured S parameters at 3.5 GHz. Table II shows the 5×5 correlation matrix of the real part of $S11$ at 1, 3.5, 6, 8.5, and 11 GHz. A valid statistical FET model must preserve these statistical relationships.

One possible method of statistically modeling the FET would be to directly model the S parameters from the measured S-parameter statistics over frequency and bias. However this solution is difficult because hundreds of parameters are needed to describe the data in the form in which they were measured. These parameters include the mean, standard deviation, densities, and correlations at each frequency and bias. Therefore, a better solution is to find a model which accurately recreates the S-parameter statistics while requiring a smaller number of parameters.

TABLE I
THE 8×8 CORRELATION MATRIX OF THE MEASURED S PARAMETERS AT 3.5 GHz: THE ORDERING IS THE REAL AND THEN IMAGINARY PARTS OF $S11$, $S12$, $S21$, AND $S22$

1.00	−0.59	−0 92	−0.29	−0.59	−0 02	0 19	−0.44
−0.59	1.00	0.60	−0.26	0.47	0.55	−0.04	0.72
−0 92	0.60	1.00	0.01	0 57	−0.02	−0.24	0.44
−0.29	−0.26	0.01	1.00	−0.10	−0 41	0.32	−0.28
−0.59	0.47	0.57	−0.10	1 00	0.58	−0.80	0.06
−0.02	0.55	−0.02	−0.41	0.58	1.00	−0.44	0.21
0.19	−0.04	−0.24	0.32	−0.80	−0.44	1.00	0 24
−0.44	0.72	0.44	−0.28	0.06	0.21	0.24	1.00

TABLE II
THE 5×5 CORRELATION MATRIX OF MEASURED REAL PART OF $S11$ AT 1, 3.5, 6, 8.5, AND 11 GHz PRESENTED IN THIS ORDER

1.00	0.97	0 95	0.86	0 76
0.97	1.00	0 99	0.94	0.85
0.95	0.99	1.00	0.97	0.90
0.86	0.94	0.97	1 00	0.98
0.76	0.85	0.90	0 98	1.00

III. THE STANDARD NONLINEAR STATISTICAL FET MODEL

An important premise of this work is that since the S parameters are the basic RF measurements, the test of a statistical model must be made against these measured statistics. The present FET modeling method starts with a set of jointly distributed random variables (S-parameter measurements) and maps them using a *nonlinear* transformation into another set of random variables, the FET model parameters (C_{gs}, g_m, R_t, etc.) [7]. In theory, to recreate the S-parameter statistics using the FET model parameters two criteria are met: 1) the mapping needs to be 1 to 1, and 2) the joint density function for the model parameters must be known. It is this second criterion that causes the problem. In practice it is difficult to determine and record the full joint density function of the model parameters. Generally only the marginal densities, correlations, and cross correlations are recorded. In general this is not sufficient to recreate the measured S-parameter densities. This is demonstrated in the following simulation study.

A. Simulation Study

To determine if the present FET model is statistically valid when using only the model parameter densities and correlations, the following study was made. Using two different assumptions for the measured S-parameter statistics, the resulting FET parameter statistics were determined. The FET parameters were then statistically modeled by retaining only the marginal densities and the correlations and cross correlations of the FET parameters. The FET's S-parameter statistics were then simulated using the derived statistics for the FET parameters. A comparison was made between the simulated and measured

TABLE III
RESULTS OF THE FET MODEL STATISTICAL SIMULATION STUDY

S-Parameter Density	Model	Number of Points	Error Range	Average Error
Gaussian and correlated	FET	1000	0.04 to 0.07	0.05
Distributed and correlated	FET	1000	0.15 to 0.69	0.34

S-parameter statistics to determine if the FET model accurately represents the measured S-parameter statistics. To measure the errors in the simulated S-parameter data an error function was developed. The densities of the simulated and the measured S parameters were represented as histograms, each with 20 bins. The error function is

$$\text{error} = \frac{\sum_{i=1}^{20} |(\text{count measured } i) - (\text{count simulated } i)|}{\text{total count}}$$

where

count measured i = the number of elements in the ith histogram bin for the measured S-parameter densities,

count simulated i = the number of elements in the ith histogram bin for the simulated S-parameter densities,

total count = the total number of elements in the histogram.

This error function represents the fraction of incorrectly placed elements in the histogram representation of the simulated S-parameter densities.

Two cases were examined using data at 6 GHz. The first case used S-parameter data that were Gaussian and correlated. The correlation is that of the 6 GHz S-parameter statistics described in Section II. The means and standard deviations for the Gaussian distribution were also those of the Section II data. This is the Gaussian and correlated case. The second case used the measured 6 GHz S-parameter densities and correlations as described in Section II. This is the distributed and correlated case. A total of 1000 simulated S-parameter measurements were used. Using the intrinsic FET model [7], the S-parameter data were mapped into FET model parameter data, thus defining seven model random variables, C_{gs}, R_i, C_{gd}, G_m, tau, C_{ds}, and g_d. Additionally an eighth variable, designated g_{dg}, was needed to make the mapping be 1 to 1. The results of this study are shown in Table III.

Table III shows that the average error in the densities of the simulated S-parameters was 5 percent when the S parameters are Gaussian and correlated. When the S-parameter densities and correlations are as measured and reported in Section II, the average error is 34 percent. The large error for the distributed and correlated case indicates the standard intrinsic FET model poorly represents the measured S-parameter statistics. An alternative statistical model is proposed in the next section.

IV. A STATISTICALLY EFFICIENT LINEAR FET MODEL

The goal of a valid statistical FET model is to accurately simulate the statistical behavior of the FET S parameters using a simple model with a small number of parameters. The following approach, based on principal component analysis, accomplishes this.

A. Principal Component Analysis

A principal component analysis (PCA) of a set of m original zero mean, unit variance random variables $(S_1, S_2, \cdots, S_m)$ creates m new *uncorrelated* random variables, the principal components (PC), $K1, K2, \cdots, Km$, with each PC being a *linear* combination of the original variables, that is,

$$K1 = b_{11}S_1 + b_{12}S_2 + \cdots + b_{1m}S_m$$
$$K2 = b_{21}S_1 + b_{22}S_2 + \cdots + b_{2m}S_m$$
$$\vdots \qquad\qquad \vdots$$
$$Km = b_{m1}S_1 + b_{m2}S_2 + \cdots + b_{mm}S_m$$

or, in matrix form, $K = BS$ [8]. The coefficients for $K1$ are chosen to make its variance as large as possible. The coefficients for $K2$ are chosen to make its variance as large as possible, subject to the restriction that $K1$ (whose variance has already been maximized) be uncorrelated with $K2$. This continues in general for all the K's. The important thing to note here is that the statistical description of the K's is simplified because they are *uncorrelated*. This paper proposes to use these K's as the statistical model parameters and to use the *linear* model

$$S = B^{-1}K$$

as the statistical FET model.

An important property of the PCA is its ability to reduce the number of K's in the model by identifying the K's which are statistically insignificant. Essentially the number of significant K's needed in the model description represents the number of independent degrees of freedom present in the S data. The example which follows determines that 13 uncorrelated principal components are needed to represent the S-parameter statistics for a 0.5 μm GaAs FET from 1 to 11 GHz at one bias. The reduced model then becomes the 13 columns of the B^{-1} matrix that are associated with the 13 significant PC's.

B. Advantages of the PCA Model

There are several advantages to using this approach. Because this is a linear model, it may be more tolerant of non-Gaussian S parameters. Certainly the linear model is much easier to implement. The PC's are uncorrelated and hence this is an advantage in the simulation. In the case of Gaussian S parameters, the statistical data are preserved exactly by simply recording the densities of the PC's. If the data need to be interpolated over frequency or bias, then the interpolation using the linear model is straightforward.

C. Disadvantages of the PCA Model

The biggest disadvantage to this model is that the PC's do not directly relate to physical process parameters. Therefore the PCA model does not scale or track with bias as the FET model does.

V. Examples of the Principal Component FET Model

Two examples of the PCA model as applied to the measured FET parameter statistics of Section II are presented in this section. First a simple PCA model is developed to model the measured FET data at 6 GHz. Then a PCA model is developed to model the measured data from 1 to 11 GHz.

A. 6 GHz PCA Model

First 1000 S parameters were simulated using the densities and correlations from the 90 measured S parameters at 6 GHz given in Section II. The model developed from these data is $K = BS$ and $S = B^{-1}K$, where $K = [K1, K2, \cdots, K8]$ is the PCA model parameter matrix, $S =$ [the real and imaginary parts of $S11$, $S12$, $S21$, and $S22$, in that order], and

$$B^{-1} = \begin{vmatrix} 0.98 & 0.00 & 0.00 & 0.01 & -0.10 & -0.04 & 0.08 & -0.13 \\ 0.97 & 0.06 & 0.06 & 0.11 & -0.02 & -0.08 & 0.09 & 0.14 \\ 0.43 & -0.21 & 0.87 & -0.07 & -0.05 & -0.01 & 0.00 & 0.00 \\ -0.95 & 0.08 & -0.20 & -0.08 & 0.03 & 0.03 & 0.19 & 0.00 \\ -0.51 & -0.23 & -0.01 & -0.15 & 0.17 & 0.80 & 0.00 & 0.00 \\ -0.46 & -0.31 & -0.06 & -0.06 & 0.81 & 0.16 & 0.00 & 0.00 \\ 0.03 & 0.95 & -0.16 & 0.16 & -0.17 & -0.12 & 0.00 & 0.00 \\ 0.44 & 0.23 & -0.08 & 0.85 & -0.05 & -0.12 & 0.00 & 0.00 \end{vmatrix}$$

The principal component analysis was performed using the S.A.S. statistical analysis package [9] using the Quartimax rotation.

This PCA model along with the 1000 S parameters was then used to determine the densities of the eight PCA model parameters. These parameters then were simulated as independent variables. Using the error function defined in Section III, the error in the measured versus simulated S parameters ranged from 0.1 to 0.18, with an average error of 0.15. This is contrasted with an average error of 0.34 when using the FET model to generate the S-parameter statistics.

B. 1 to 11 GHz PCA Model

This example uses the measured S-parameter data from 90 GaAs FET's measured at 1, 3.5, 6, 8.5, and 11 GHz at $I_d = I_{dss}$. The S-parameter data are in real and imaginary form. Since there are four S parameters, there are eight data points for each frequency. The data for each FET were arranged into a vector of 40 elements, eight elements for each frequency times the five frequencies at which the FET's were measured. Therefore, the S-parameter data consisted of 90 vectors of length 40. Each random variable was normalized to zero mean and unit variance. These data represent samples from 40 random variables, and consequently the joint density function has dimension of 40. The densities for the 40 S parameters are not in general Gaussian or "bell-shaped," as noted in Section II.

A principal component analysis was initiated on these data using the S.A.S. statistical analysis package [9] using the Quartimax rotation. This analysis gave a 40×40 coefficient matrix, B. The S.A.S. analysis indicated that there were 13 statistically significant principal components. The appropriate 13 columns of B^{-1} were identified. The reduced B^{-1} matrix is partially shown below:

$$B^{-1\prime} = \begin{vmatrix} -0.962 & 0.008 & -0.139 & 0.117 & -0.004 & \cdots & 0.004 \\ -0.973 & 0.081 & -0.001 & 0.077 & -0.001 & \cdots & -0.004 \\ -0.575 & -0.150 & 0.019 & -0.787 & -0.015 & \cdots & -0.002 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0.687 & -0.632 & 0.295 & 0.042 & 0.001 & \cdots & -0.021 \end{vmatrix}$$

To test the model the 13 uncorrelated PC's were simulated according to their marginal densities, which were obtained using the measured S-parameter data and the linear model $K = BS$. Fig. 2 shows the densities for the first four PC's (K_1, K_2, K_3, and K_4). The PC's are uncorrelated but are assumed independent in the simulation. A sample set of 500 points was generated. The simulated S parameters were then created from the reduced order model $S = B^{-1\prime}K$, where $B^{-1\prime}$ is the 40×13 reduced parameter matrix.

A comparison was then made between the measured S-parameter densities and correlations and the simulated S-parameter densities and correlations. The error for the S-parameter densities ranged from 0.11 to 0.25 with an average error of 0.18. Table IV shows the 8×8 correlation matrix of the simulated S parameters at 3.5 GHz, and Table V shows the 5×5 correlation matrix of the simulated real part of $S11$ at 1, 3.5, 6, 8.5, and 11 GHz. In general the measured and simulated data matched well, as is evidenced by an average error of 0.18 and a comparison of Tables I, II, IV, and V.

VI. Simulation Using the PCA Model

After the PCA model has been generated, simulation of the S parameters is straightforward. Fig. 3 shows a simplified diagram for the generation of the simulated S param-

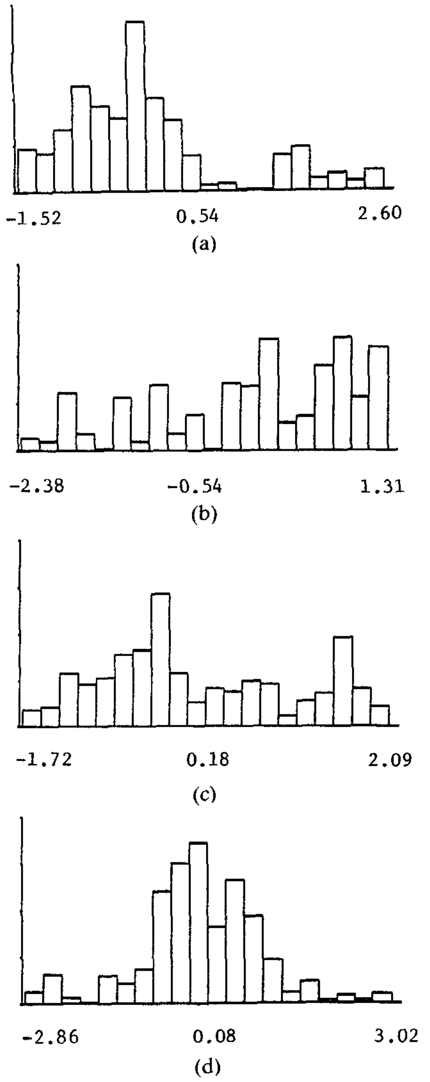

Fig. 2. Densities for the first four principal components for the 1 to 11 GHz PCA model: (a) K_1, (b) K_2, (c) K_3, and (d) K_4.

TABLE IV
THE 8 × 8 CORRELATION MATRIX OF THE SIMULATED S PARAMETERS AT 3.5 GHz: ORDERING IS REAL AND IMAGINARY PARTS OF $S11$, $S12$, $S21$, $S22$

1.00	−0.59	−0.93	−0.35	−0.60	0.01	0.16	−0.42
−0.59	1.00	0.60	−0.20	0.45	0.54	0.02	0.70
−0.93	0.60	1.00	0.10	0.57	−0.05	−0.19	0.40
−0.35	−0.20	0.10	1.00	−0.05	−0.43	0.28	−0.25
−0.60	0.45	0.57	−0.05	1.00	0.57	−0.80	0.03
0.01	0.54	−0.05	−0.43	0.57	1.00	−0.42	0.21
0.16	0.02	−0.19	0.28	−0.80	−0.42	1.00	0.28
−0.42	0.70	0.40	−0.25	0.03	0.21	0.28	1.00

TABLE V
THE 5 × 5 CORRELATION MATRIX OF SIMULATED REAL PART OF $S11$ AT 1, 3.5, 6, 8.5, AND 11 GHz, IN THIS ORDER

1.00	0.97	0.95	0.87	0.77
0.97	1.00	1.00	0.94	0.86
0.95	1.00	1.00	0.97	0.90
0.87	0.94	0.97	1.00	0.98
0.77	0.86	0.90	0.98	1.00

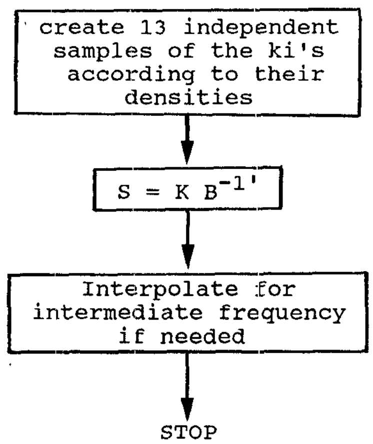

Fig. 3. Simulation diagram for the calculation of the simulated S parameters using the PCA model

eters. An important point is that all the S-parameter data over frequency are generated by one pass through this simulation algorithm.

VII. A STATISTICAL MODEL FOR $I_d = I_{dss}/2$

The S-parameter data were also analyzed for the bias $I_d = I_{dss}/2$. A comparison was made between the S-parameter statistics at the two biases, and they appear to have little in common. It is speculated that the PC's for the $I_d = I_{dss}$ bias will not be sufficient to describe the statistics at $I_d = I_{dss}/2$. Therefore the number of model parameters will approximately double if the model is to be valid at two bias points. More work needs to be done in this area to overcome this apparent difficulty.

VIII. CONCLUSIONS

There were two main goals of this work. One was to examine the standard FET model and determine if it is statistically valid. Using a simulation study the average error in the S-parameter densities, as defined by our error function, is 0.05 for the Gaussian and correlated case and 0.34 for the distributed and correlated case. The simulation used only the model parameter correlations and densities. In the case of non-Gaussian S parameters, the FET model is statistically poor. Since measured S-parameter data are shown to be non-Gaussian, a new statistical modeling technique is desired.

The second objective of this work was to propose a new linear statistical model that does not depend as heavily upon the Gaussian assumption. A modeling technique based on principal component analysis of the measured S-parameter data was proposed and demonstrated. This model used 13 uncorrelated parameters (PC's) to describe the FET statistics from 1 to 11 GHz at one bias. In addition the model is linear. The S-parameter means and standard deviations must also be recorded as the data were normalized prior to analysis. The average errors for the simulated S parameters were 0.15 for the 6 GHz model and 0.18 for the 1 to 11 GHz model.

In summary the PCA model requires, for each frequency of interest, S-parameter means, standard deviations, the coefficients of the $B^{-1\prime}$ matrix, and the densities for the principal components. These data form the entire statistical model. Note that the present S-parameter data supplied by the foundry constitute a subset of these data.

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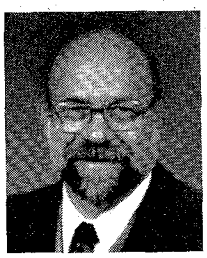

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